AQA A-Level Physics: Electric Fields – Complete Calculation Solutions

Praneel Physics

1. Calculate the force between two $+5\,\mu$C charges 3 cm apart. **(P)**

Working and Answer:

$$F = \frac{kq_1q_2}{r^2} = \frac{(8.99\times10^9)(5\times10^{-6})^2}{(0.03)^2}$$
$$= \frac{224.75}{0.0009} = 249{,}722\,\text{N}$$
$$= \boxed{2.50\times10^5\,\text{N (repulsive)}}$$

2. Determine the electric field 2 cm from a $-10\,\text{nC}$ point charge. **(P)**

Working and Answer:

$$E = \frac{k|q|}{r^2} = \frac{(8.99 \times 10^9)(10 \times 10^{-9})}{(0.02)^2}$$
$$= \frac{89.9}{0.0004} = 224{,}750\,\text{N/C}$$
$$= \boxed{2.25 \times 10^5\,\text{N/C (toward charge)}}$$

3. Calculate the potential at 5 cm from a +8 nC charge. **(P)**

Working and Answer:

$$V = \frac{kq}{r} = \frac{(8.99 \times 10^{9})(8 \times 10^{-9})}{0.05}$$
$$= \frac{71.92}{0.05} = 1{,}438\,\mathrm{V}$$
$$= \boxed{1.44\,\mathrm{kV}}$$

4. A $0.5\,\mu$C charge experiences $10\,$N force in an electric field. Find the field strength. **(P)**

Working and Answer:

$$E = \frac{F}{q} = \frac{10}{0.5 \times 10^{-6}}$$
$$= 2.0 \times 10^{7}\,\text{N/C}$$
$$= \boxed{2.0 \times 10^{7}\,\text{N/C}}$$

5. Two parallel plates are 2 mm apart with 100 V across them. Find the electric field strength. **(P)**

Working and Answer:

$$
\begin{aligned}
E &= \frac{V}{d} = \frac{100}{0.002} \\
&= 50{,}000\,\text{N/C} \\
&= \boxed{50\,\text{kN/C}}
\end{aligned}
$$

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6. Calculate the work done moving a $+3\,\mu$C charge through 12 V. **(P)**

Working and Answer:

$$\begin{aligned} W = qV &= (3 \times 10^{-6})(12) \\ &= 36 \times 10^{-6}\,\text{J} \\ &= \boxed{36\,\mu\text{J}} \end{aligned}$$

7. A $+2\,\mu$C charge moves 25 cm in a uniform electric field of 400 N/C. Calculate the work done. **(PP)**

Working and Answer:

$$\begin{aligned} F &= qE = (2 \times 10^{-6})(400) = 8 \times 10^{-4}\,\text{N} \\ W &= Fd = (8 \times 10^{-4})(0.25) = 2 \times 10^{-4}\,\text{J} \\ &= \boxed{0.2\,\text{mJ}} \end{aligned}$$

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8. Find the acceleration of an electron ($m_e = 9.11 \times 10^{-31}\,\text{kg}$) in a $500\,\text{N/C}$ electric field. **(PP)**

Working and Answer:

$$
\begin{aligned}
F &= qE = (1.6 \times 10^{-19})(500) = 8 \times 10^{-17}\,\text{N} \\
a &= \frac{F}{m} = \frac{8 \times 10^{-17}}{9.11 \times 10^{-31}} \\
&= 8.78 \times 10^{13}\,\text{m/s}^2 \\
&= \boxed{8.8 \times 10^{13}\,\text{m/s}^2}
\end{aligned}
$$

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9. Two charges ($+4\,\mu$C and $-6\,\mu$C) are 8 cm apart. Find the force between them. **(PP)**

Working and Answer:

$$
\begin{aligned}
F &= \frac{k|q_1 q_2|}{r^2} = \frac{(8.99 \times 10^9)(4 \times 10^{-6})(6 \times 10^{-6})}{(0.08)^2} \\
&= \frac{215.76}{0.0064} = 33{,}712.5\,\text{N} \\
&= \boxed{33.7\,\text{N (attractive)}}
\end{aligned}
$$

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10. A 3 cm diameter sphere carries 50 nC charge. Find the electric field at its surface. **(PP)**

Working and Answer:

$$E = \frac{k|q|}{r^2} = \frac{(8.99 \times 10^9)(50 \times 10^{-9})}{(0.015)^2}$$

$$= 1.998 \times 10^6 \text{ N/C}$$

$$= \boxed{2.0 \times 10^6 \text{ N/C}}$$

11. Calculate the final speed of a proton ($m_p = 1.67 \times 10^{-27}$ kg) accelerated through 1 kV. **(PP)**

Working and Answer:

$$KE = qV = (1.6 \times 10^{-19})(1000) = 1.6 \times 10^{-16}\,\text{J}$$
$$v = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{3.2 \times 10^{-16}}{1.67 \times 10^{-27}}}$$
$$= 4.38 \times 10^{5}\,\text{m/s}$$
$$= \boxed{438\,\text{km/s}}$$

12. Find the potential energy of a $+5\,\mu$C and $-3\,\mu$C pair separated by 4 cm. **(PP)**

Working and Answer:

$$U = \frac{kq_1q_2}{r} = \frac{(8.99 \times 10^9)(5 \times 10^{-6})(-3 \times 10^{-6})}{0.04}$$
$$= -3.371\,\text{J}$$
$$= \boxed{-3.37\,\text{J}}$$

13. A $+3\,\mu$C charge moves from 200 V to 800 V. If its mass is 0.5 g, calculate its final kinetic energy and speed. **(PPP)**

Working and Answer:

Solution:

$$\text{Step 1: Work done} = q\Delta V = (3 \times 10^{-6})(600) = 1.8\,\text{mJ}$$

$$\text{Step 2: Kinetic energy} = \text{Work} = 1.8\,\text{mJ}$$

$$\text{Step 3: Speed} = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{3.6 \times 10^{-3}}{0.5 \times 10^{-3}}}$$

$$= \sqrt{7.2} = 2.68\,\text{m/s}$$

$$= \boxed{2.68\,\text{m/s}}$$

14. Three charges form a right triangle: $+2\,\mu\text{C}$ at $(0,0)$, $+3\,\mu\text{C}$ at $(4\,\text{cm},0)$, and $-5\,\mu\text{C}$ at $(0,3\,\text{cm})$. Find the net force on the $+2\,\mu\text{C}$ charge. **(PPP)**

Working and Answer:

Solution:

$$\text{Step 1: Calculate } F_{12} = \frac{(8.99\times10^{9})(2\times10^{-6})(3\times10^{-6})}{(0.04)^2} = 33.71\,\text{N (right)}$$

$$\text{Step 2: Calculate } F_{13} = \frac{(8.99\times10^{9})(2\times10^{-6})(5\times10^{-6})}{(0.03)^2} = 99.89\,\text{N (up)}$$

$$\text{Step 3: Net force} = \sqrt{33.71^2+99.89^2} = 105.4\,\text{N at } 71.6^\circ$$

$$= \boxed{105\,\text{N at } 71.6^\circ \text{ above +x-axis}}$$

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15. A parallel plate capacitor has 1 cm spacing and 200 V across it. A $+1\,\mu$C charge (mass = 2 mg) is released from rest at the positive plate. Calculate its speed at the negative plate. **(PPP)**

Working and Answer:

Solution:

$$\text{Step 1: Field strength} = \frac{200}{0.01} = 20{,}000\,\text{N/C}$$

$$\text{Step 2: Acceleration} = \frac{qE}{m} = \frac{(1\times10^{-6})(20{,}000)}{2\times10^{-6}} = 10{,}000\,\text{m/s}^2$$

$$\text{Step 3: Final speed} = \sqrt{2ad} = \sqrt{2(10{,}000)(0.01)} = \sqrt{200}$$

$$= \boxed{14.1\,\text{m/s}}$$

16. Calculate the capacitance of two $0.5\,\text{m}^2$ plates separated by $2\,\text{mm}$ with a dielectric ($\kappa = 4$) between them. **(PPP)**

Working and Answer:

Solution:

$$\text{Step 1: Without dielectric} = \frac{\epsilon_0 A}{d} = \frac{(8.85 \times 10^{-12})(0.5)}{0.002} = 2.21\,\text{nF}$$
$$\text{Step 2: With dielectric} = \kappa C_0 = 4(2.21\,\text{nF})$$
$$\text{Step 3: Final capacitance} = \boxed{8.84\,\text{nF}}$$

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17. A $100\,\mu\text{F}$ capacitor is charged to $50\,\text{V}$. Calculate the energy stored and the charge on each plate. **(PPP)**

Working and Answer:

Solution:

$$\begin{aligned}
\text{Step 1: Energy} &= \frac{1}{2}CV^2 = \frac{1}{2}(100 \times 10^{-6})(50)^2 = 125\,\text{mJ} \\
\text{Step 2: Charge} &= CV = (100 \times 10^{-6})(50) = 5\,\text{mC} \\
\text{Step 3: Results} &= \boxed{125\,\text{mJ energy}}, \boxed{5\,\text{mC charge}}
\end{aligned}$$

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18. A dipole ($q = \pm 10\,\text{nC}$, $d = 1\,\text{mm}$) is in a $500\,\text{N/C}$ field. Calculate the maximum torque and potential energy difference between aligned and anti-aligned states. **(PPP)**

Working and Answer:

Solution:

$$\text{Step 1: Dipole moment} = qd = (10 \times 10^{-9})(0.001) = 1 \times 10^{-11}\,\text{C}\cdot\text{m}$$
$$\text{Step 2: Maximum torque} = pE = (1 \times 10^{-11})(500) = \boxed{5\,\text{nN}\cdot\text{m}}$$
$$\text{Step 3: Energy difference} = 2pE = \boxed{10\,\text{nJ}}$$

Praneel Physics

19. Derive the electric field at distance z from a charged ring ($Q = 50\,\text{nC}$, $R = 5\,\text{cm}$) and calculate at $z = 3\,\text{cm}$. **(PPPP)**

Working and Answer:

Solution:

$$\text{Step 1: Linear charge density} = \frac{Q}{2\pi R}$$

$$\text{Step 2: Differential field components} = \frac{k\lambda dl}{(z^2 + R^2)}\cos\theta$$

$$\text{Step 3: Integrate around ring} = \frac{kQz}{(z^2 + R^2)^{3/2}}$$

$$\text{Step 4: Calculate at z=3 cm} = \frac{(8.99 \times 10^9)(50 \times 10^{-9})(0.03)}{(0.03^2 + 0.05^2)^{3/2}}$$

$$= \boxed{1.08 \times 10^5\,\text{N/C}}$$

20. Four $+2\,\mu$C charges are arranged in a square (side 10 cm). Calculate the total potential energy of the system. **(PPPP)**

Working and Answer:

Solution:

$$\text{Step 1: Single pair energy} = \frac{k(2\times10^{-6})^2}{0.10} = 0.3596\,\text{J}$$

$$\text{Step 2: Adjacent pairs (4)} = 4(0.3596) = 1.438\,\text{J}$$

$$\text{Step 3: Diagonal pairs (2)} = 2\left(\frac{k(2\times10^{-6})^2}{0.10\sqrt{2}}\right) = 0.5086\,\text{J}$$

$$\text{Step 4: Total energy} = 1.438 + 0.5086 = \boxed{1.947\,\text{J}}$$

Praneel Physics

21. A $+5\,\mu$C charge moves from $(2\,\text{m}, 0)$ to $(5\,\text{m}, 4\,\text{m})$ in $\mathbf{E} = 300\,\hat{\imath}\,\text{N/C}$. Calculate the work done. **(PPPP)**

Working and Answer:

Solution:

$$\text{Step 1: Displacement in x-direction} = 5 - 2 = 3\,\text{m}$$
$$\text{Step 2: Force in x-direction} = qE = (5 \times 10^{-6})(300) = 1.5\,\text{mN}$$
$$\text{Step 3: Work along x-axis} = F_x \Delta x = (1.5 \times 10^{-3})(3)$$
$$\text{Step 4: Total work} = \boxed{4.5\,\text{mJ}}$$

Praneel Physics

22. Derive the capacitance of an isolated conducting sphere (radius R) and calculate for $R = 6\,\text{cm}$. **(PPPP)**

Working and Answer:

Solution:

$$\text{Step 1: Potential of sphere} = \frac{kQ}{R}$$
$$\text{Step 2: Capacitance definition} = \frac{Q}{V} = \frac{Q}{kQ/R}$$
$$\text{Step 3: Simplify} = 4\pi\epsilon_0 R$$
$$\text{Step 4: Calculate} = 4\pi(8.85 \times 10^{-12})(0.06) = \boxed{6.67\,\text{pF}}$$

23. A parallel plate capacitor ($C = 10\,\mu$F) has its energy reduced to 1/5 original when a dielectric is inserted at constant voltage. Find the dielectric constant. **(PPPP)**

Working and Answer:

Solution:

$$\text{Step 1: Original energy} = \frac{1}{2}C_0V^2$$

$$\text{Step 2: New energy} = \frac{1}{2}\kappa C_0V^2 = \frac{1}{5}U_0$$

$$\text{Step 3: Ratio} = \frac{1}{\kappa} = \frac{1}{5}$$

$$\text{Step 4: Dielectric constant} = \boxed{5}$$

24. Calculate the electric field 3 cm from an infinite line charge with $\lambda = 2\,\mu\text{C/m}$. **(PPPP)**

Working and Answer:

Solution:

$$\text{Step 1: Gauss's law setup} = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$
$$\text{Step 2: Cylindrical symmetry} = E(2\pi r L) = \frac{\lambda L}{\epsilon_0}$$
$$\text{Step 3: Solve for E} = \frac{\lambda}{2\pi\epsilon_0 r}$$
$$\text{Step 4: Calculate} = \frac{2 \times 10^{-6}}{2\pi(8.85 \times 10^{-12})(0.03)} = \boxed{1.20 \times 10^{6}\ \text{N/C}}$$

Praneel Physics

25. Derive the capacitance per unit length of coaxial cable (inner radius $a = 1\,\text{mm}$, outer $b = 4\,\text{mm}$, $\kappa = 3$) and calculate the value. **(PPPPP)**

Working and Answer:

Solution:

$$\text{Step 1: Gauss's law for inner conductor} = E = \frac{\lambda}{2\pi\kappa\epsilon_0 r}$$

$$\text{Step 2: Potential difference} = \int_a^b E\,dr = \frac{\lambda}{2\pi\kappa\epsilon_0}\ln(b/a)$$

$$\text{Step 3: Capacitance definition} = \frac{\lambda L}{V} = \frac{2\pi\kappa\epsilon_0 L}{\ln(b/a)}$$

$$\text{Step 4: Per unit length} = \frac{2\pi\kappa\epsilon_0}{\ln(b/a)}$$

$$\text{Step 5: Calculate} = \frac{2\pi(3)(8.85\times10^{-12})}{\ln(4)} = \boxed{151\,\text{pF/m}}$$

26. Using Gauss's law, derive the electric field inside a uniformly charged sphere ($\rho = 3\,\mu\text{C/m}^3$, $R = 10\,\text{cm}$) at $r = 4\,\text{cm}$ from center. **(PPPPP)**

Working and Answer:

Solution:

$$\text{Step 1: Enclosed charge} = \rho\left(\frac{4}{3}\pi r^3\right) = (3\times10^{-6})\left(\frac{4}{3}\pi(0.04)^3\right)$$

$$\text{Step 2: Gauss's law} = \oint \mathbf{E}\cdot d\mathbf{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

$$\text{Step 3: Symmetry} = E(4\pi r^2) = \frac{\rho\frac{4}{3}\pi r^3}{\epsilon_0}$$

$$\text{Step 4: Solve for E} = \frac{\rho r}{3\epsilon_0}$$

$$\text{Step 5: Calculate} = \frac{(3\times10^{-6})(0.04)}{3(8.85\times10^{-12})} = \boxed{4.52\times10^3\,\text{N/C}}$$

27. An electron enters parallel plates (5 cm long, 1 cm gap, 100 V) at 1×10^6 m/s. Calculate the vertical deflection. **(PPPPP)**

Working and Answer:

Solution:

$$\begin{aligned}
\text{Step 1: Field strength} &= \frac{100}{0.01} = 10{,}000\,\text{N/C} \\
\text{Step 2: Acceleration} &= \frac{eE}{m} = \frac{(1.6 \times 10^{-19})(10^4)}{9.11 \times 10^{-31}} = 1.76 \times 10^{15}\,\text{m/s}^2 \\
\text{Step 3: Time in field} &= \frac{0.05}{1 \times 10^6} = 50\,\text{ns} \\
\text{Step 4: Vertical displacement} &= \frac{1}{2}at^2 = \frac{1}{2}(1.76 \times 10^{15})(50 \times 10^{-9})^2 \\
&= \boxed{2.20\,\text{mm}}
\end{aligned}$$

28. Derive the potential due to a dipole and calculate at point $(3\,\text{cm}, 4\,\text{cm})$ from center $(p = 2 \times 10^{-10}\,\text{C} \cdot \text{m})$ oriented along x-axis. **(PPPPP)**

Working and Answer:

Solution:

$$\text{Step 1: Dipole potential formula} = \frac{k\mathbf{p} \cdot \mathbf{r}}{r^3}$$

$$\text{Step 2: Position vector} = (0.03, 0.04)\,\text{m}, r = 0.05\,\text{m}$$

$$\text{Step 3: Angle calculation} = \cos\theta = \frac{0.03}{0.05} = 0.6$$

$$\text{Step 4: Potential} = \frac{(8.99 \times 10^9)(2 \times 10^{-10})(0.6)}{(0.05)^2}$$

$$= \boxed{43.2\,\text{mV}}$$

29. Show the energy density in electric field is $\frac{1}{2}\epsilon_0 E^2$ and calculate for $E = 5\,\text{MV/m}$. **(PPPPP)**

Working and Answer:

Solution:

$$\begin{aligned}
\text{Step 1: Capacitor energy} &= \frac{1}{2}CV^2 \\
\text{Step 2: Express in terms of E} &= \frac{1}{2}\left(\frac{\epsilon_0 A}{d}\right)(Ed)^2 \\
\text{Step 3: Energy density} &= \frac{U}{Ad} = \frac{1}{2}\epsilon_0 E^2 \\
\text{Step 4: Calculate} &= \frac{1}{2}(8.85 \times 10^{-12})(5 \times 10^6)^2 \\
&= \boxed{110.6\,\text{J/m}^3}
\end{aligned}$$

30. Derive the oscillation frequency of a dipole ($p = 5 \times 10^{-10}\,\mathrm{C \cdot m}$, $I = 2 \times 10^{-26}\,\mathrm{kg \cdot m^2}$) in $E = 1\,\mathrm{kN/C}$. **(PPPPP)**

Working and Answer:

Solution:

$$\text{Step 1: Torque equation} = -pE\sin\theta \approx -pE\theta$$

$$\text{Step 2: Angular acceleration} = \alpha = \frac{\tau}{I} = -\frac{pE}{I}\theta$$

$$\text{Step 3: SHM equation} = \frac{d^2\theta}{dt^2} + \frac{pE}{I}\theta = 0$$

$$\text{Step 4: Angular frequency} = \omega = \sqrt{\frac{pE}{I}}$$

$$\text{Step 5: Calculate} = \sqrt{\frac{(5 \times 10^{-10})(1000)}{2 \times 10^{-26}}} = \boxed{1.58 \times 10^{10}\,\mathrm{rad/s}}$$